Exam 3 Review

Ex: Sketch a continuous function

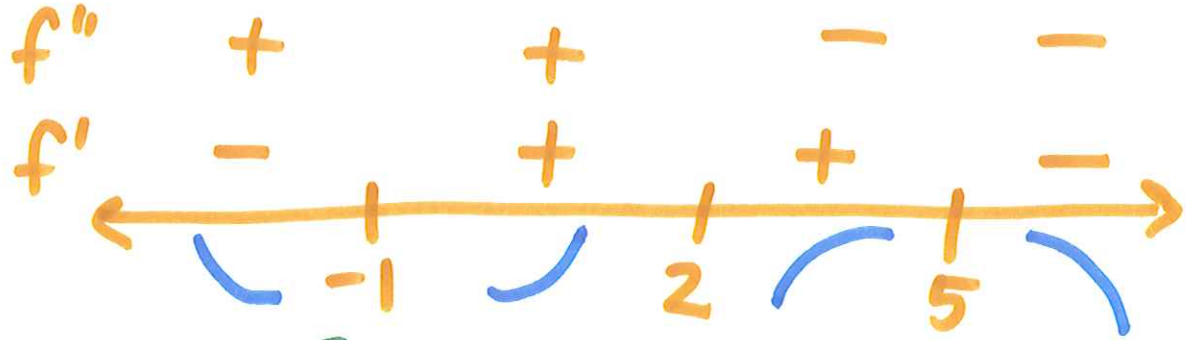
Satisfying $f'(x) < 0$ for $x < -1$ and $x > 5$

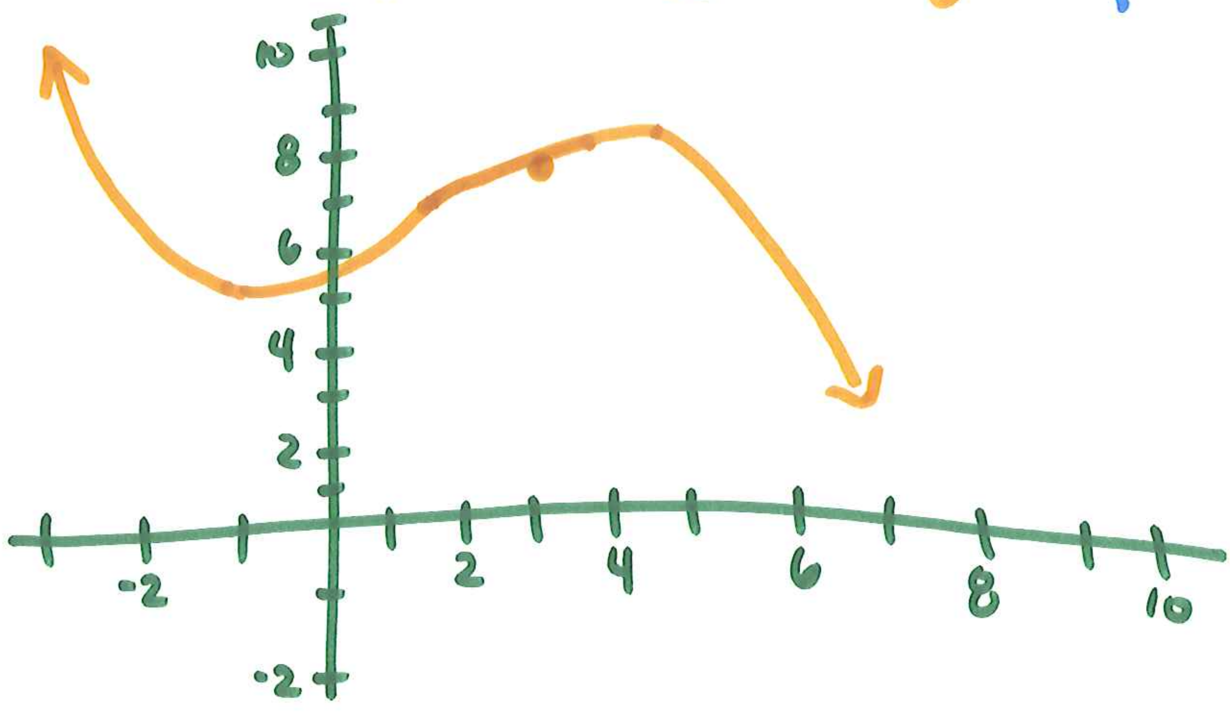
$f'(x) > 0$ for $-1 < x < 5$

$f''(x) > 0$ for $x < 2$

$f''(x) < 0$ for $x > 2$

$f(3) = 8$





Ex: let $f'(x) = (x+4)(2-x)$

a) When is f increasing / decreasing?

$f'(x) = 0$ when $x = -4, 2$

f': $-$ | $+$ | $-$

-4 2

f increasing $(-4, 2)$

f decreasing $(-\infty, -4) \cup (2, \infty)$

b) When is f concave up / down?

$f'(x) = 2x + 8 - x^2 - 4x = 8 - 2x - x^2$

$f''(x) = -2 - 2x$

$f''(x) = 0$ when $x = -1$

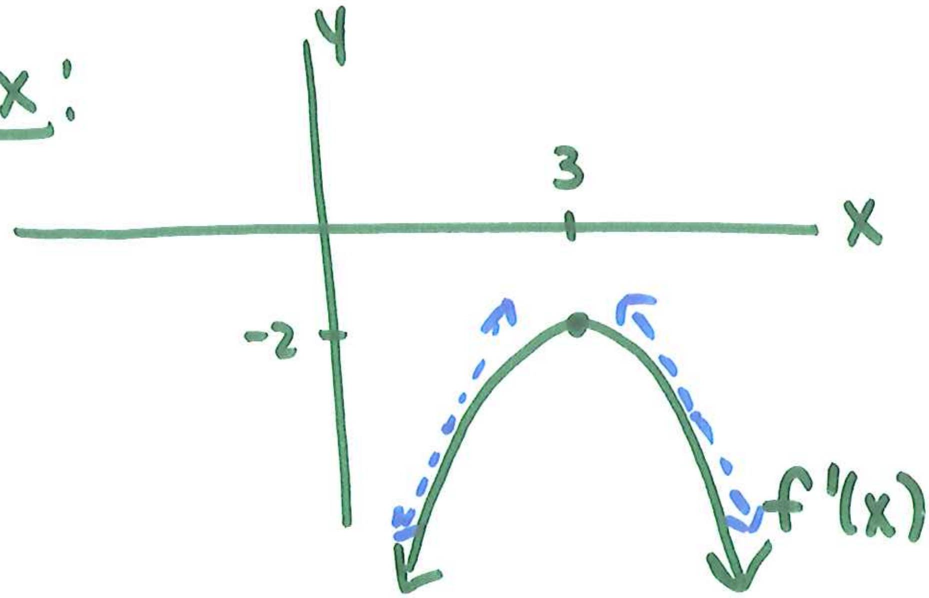
f'': $+$ | $-$

-1

f concave up $(-\infty, -1)$

f concave down $(-1, \infty)$

Ex:



Where is f(x) increasing and decreasing?
Where is f(x) concave up and concave down?

$f'(x) < 0$ for all x → f(x) always decreasing
f(x) never increasing

$f''(x) > 0$ ($f'(x)$ has "+" sloped tangent lines when $x < 3$ → f(x) concave up
$(-\infty, 3)$

$f''(x) < 0$ ($f'(x)$ has "−" sloped tangent lines when $x > 3$ → f(x) concave down
$(3, \infty)$

Ex: x and y are positive numbers, $x \cdot y = 33$. What is the minimum possible sum $x+y$?

$x, y > 0$

$x \cdot y = 33 \rightarrow y = \frac{33}{x}$

then $x+y = x + \frac{33}{x} = f(x)$

if $f(x) = x + 33x^{-1}$

then $f'(x) = 1 - 33x^{-2} = 1 - \frac{33}{x^2}$

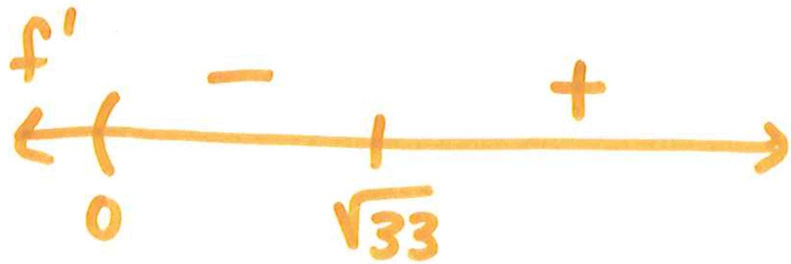
$1 - \frac{33}{x^2} = 0$

$1 = \frac{33}{x^2}$

$x^2 = 33$

$x = \pm\sqrt{33}$

$-\sqrt{33}$ not in interval $(x>0)$



min occurs at $x = \sqrt{33}$

$y = 33/\sqrt{33} = \sqrt{33}$

So min sum is

$\sqrt{33} + \sqrt{33} = \boxed{2\sqrt{33}}$

Ex: Evaluate $\underbrace{3\cdot 1 + 3\cdot 4 + 3\cdot 9 + 3\cdot 16 + \dots + 3\cdot 1600}_{3(1+4+9+16+\dots+1600)}$

$= 3k^2$

$$\sum_{k=1}^{40} 3k^2 = 3\sum_{k=1}^{40} k^2 = 3\left(\frac{40(41)(81)}{6}\right)$$

$$= \boxed{66,420}$$

Ex: You estimate $\int_4^{20} f(x)\,dx = \sum_{k=1}^{n} f\left(4 + k\cdot\frac{A}{n}\right)\cdot\frac{A}{n}$

What is A?

$$\Delta x = \frac{20-4}{n} = \frac{16}{n}$$

$$x_k = 4 + k\left(\frac{16}{n}\right)$$

thus $\boxed{A = 16}$